The Chi Squared Test
卡方检验

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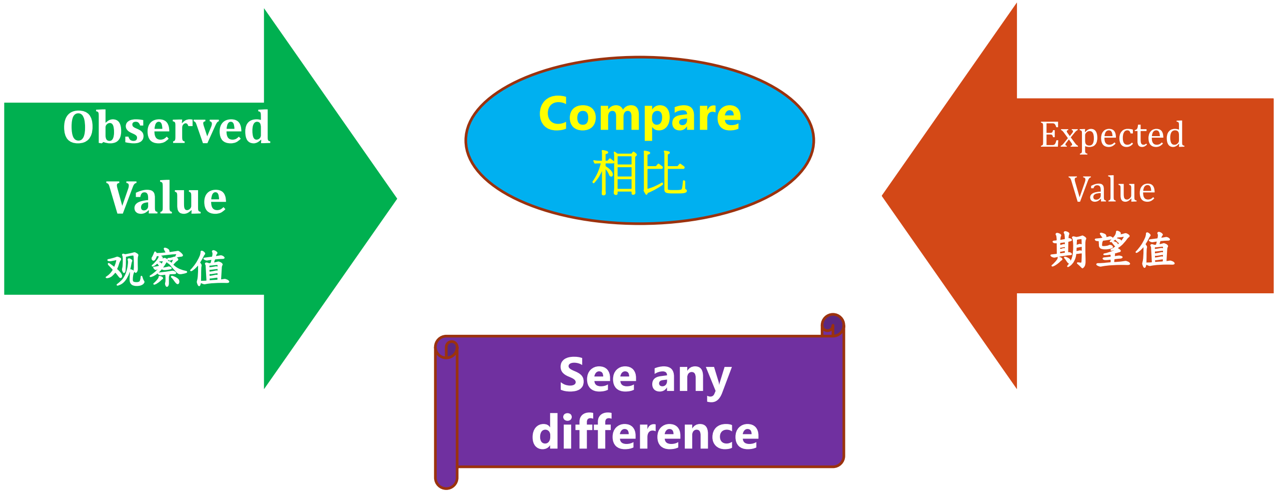
Chi Square Test 卡方检验

- **Test for Proportion 概率测试**
- Understand and analyze the **relationship on Frequency/ counts** between two categorical variables
  理解和分析两个分类变量之间频率/计数的关系



Observed Value 观察值

Compare 相比

Expected Value **期望值**

See any difference

Test of proportion 概率测试

- If Drug A and Drug B have same effect
- 如果药物A和药物B具有相同的效果
- We can get the expected values for the 4 boxes by the following methods
- 我们可以通过以下方法得到4个框的期望值

	Death	Alive	
Drug A	a	b	a+b
Drug B	c	d	c+d
	a+c	b+d	a+b+c+d (all)

$$\textbf{Box expected value} = \frac{\textbf{Row total x column total}}{\textbf{Overall total}}$$

e.g. expected value for the box with observed value a = (a+b) (a+c) / all

Chi Square Test 卡方检验

- Measure **the association** between two categorical variables

  检定两组类别变量的关联性

- Examples of categorical variables with only two categories: Gender (Female and Male), Dead or Alive, Age group
- 只有两个类别的分类变量示例：性别（女性和男性）、死或生、年龄组

  Dichotomous 二分法

- Use the Chi Square distributions and critical value to **accept or reject our hypothesis**
- 使用卡方分布和临界值来接受或拒绝我们的假设

  假设检定:

H_0: A 变项与 B 变项之间没有关联性

H_1: A 变项与 B 变项之间具有关联性

Assumption 前提假设

1. 所有的变项为**类别变项(categorical variable)**

2. 样本须为**独立变项(Independent variable)**

即是：第一组的样本不影响第二组的样本；
第二组的样本也不影响第一组

- 3. 每一检定细格(cell)内的数据应该设为频率(Frequency)或次数(count)，而不是百分比或是经过转换之数据。

Using 2x2 contingency table to explain

Observed value 观察值

	Dead	Alive	
Drug A	a 30	b 40	70 (Row Total) a+b
Drug B	C 60	d 70	130 (Row Total) c+d
	90 (Column Total) a+c	110 (Column Total) b+d	200 Overall Total (a+b+c+d)

How to calculate the expected value 期望值

For each group a/b/c/d

$$\frac{\text{Row total x Column Total}}{\text{Overall total}}$$

For Box A

a

$$\frac{(a+b) \text{ x } (a+c)}{a+b+c+d}$$

$= \underline{(30+40) \text{ x } (30+60)} \;/\; (30+40+60+70)$

$= 31.5$

For Box B

b

$$\frac{(a+b) \text{ x } (b+d)}{a+b+c+d}$$

$$= \frac{(30+40) \text{ x } (40+70)}{200}$$

$= 38.5$

	Dead	Alive	
Drug A	a **30**	b **40**	**70 (Row Total)** **a+b**
Drug B	C **60**	d **70**	**130 (Row Total)** **c+d**
	90 (Column Total) a+c	110 (Column Total) b+d	**200** **Overall Total** **(a+b+c+d)**

How to calculate the expected value 预期值

For each group a/b/c/d

$$\frac{\text{Row total x Column Total}}{\text{Overall total}}$$

	Dead	Alive	
Drug A	a 30	b 40	70 (Row Total) a+b
Drug B	C 60	d 70	130 (Row Total) c+d
	90 (Column Total) a+c	110 (Column Total) b+d	200 Overall Total (a+b+c+d)

For Box C

c

$$\frac{(c+d) \times (a+c)}{a+b+c+d} = \frac{130 \times 90}{200} = 58.5$$

For Box D

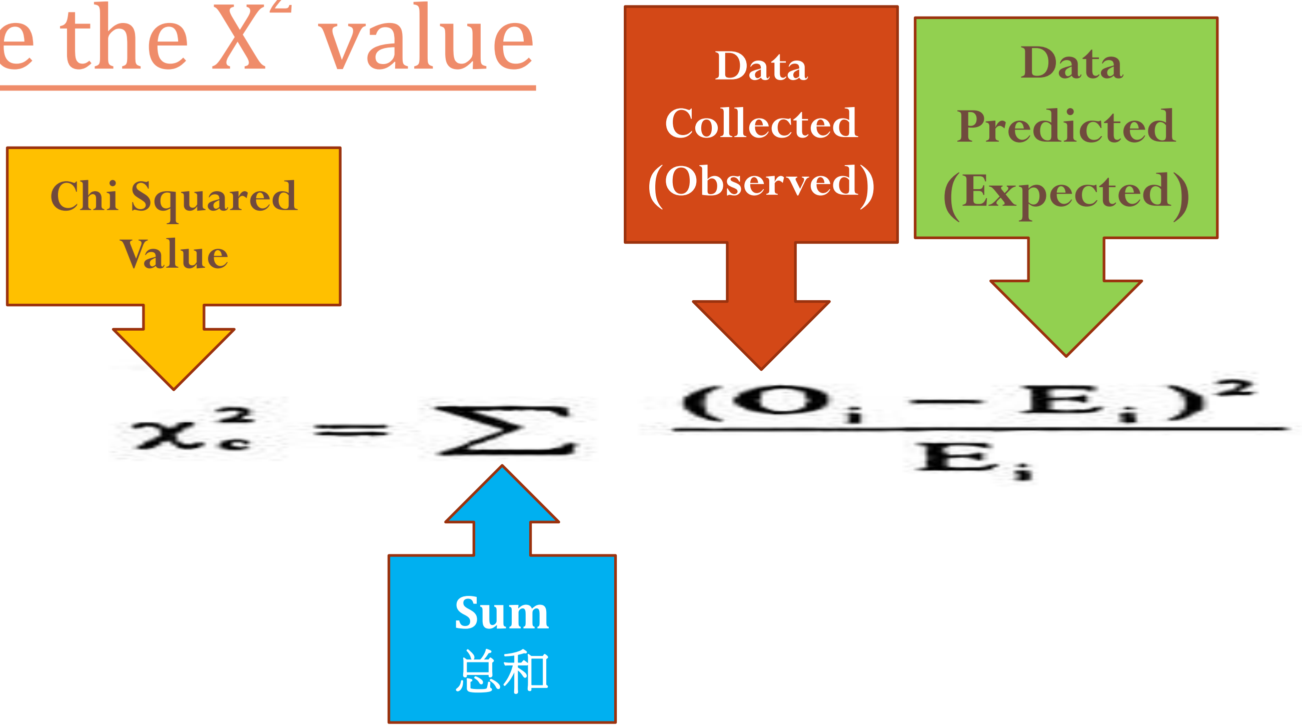
d

$$\frac{(c+d) \times (b+d)}{a+b+c+d} = \frac{130 \times 110}{200} = 71.5$$

See the difference between the observed values and the expected values
查看观察值和期望值之间的差异

	Observed观察值	Expected预期值	Difference差异
Drug A Dead	a **30**	31.5	**- 1.5**
Drug A Alive	b **40**	38.5	**1.5**
Drug B Dead	**C** **60**	58.5	**1.5**
Drug B Alive	**D** **70**	71.5	**-1.5**

Calculate the X^2 value



Chi Squared Value

Data Collected (Observed)

Data Predicted (Expected)

$$\chi_c^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

Sum 总和

X^2 = (2.25/31.5) +(2.25/38.5) + (2.25/58.5) + (2.25/71.5)

= 0.071 + 0.058+ 0.038 + 0.031

= 0.198

degree of freedom (df) 自由度

- 统计学上的**自由度**，是指当以样本的统计量来估计母体的参数时，样本中独立或能自由变化的数据的个数，称为该统计量的自由度。

维基百科

- 指的是计算某一统计量时，取值不受限制的变量个数。

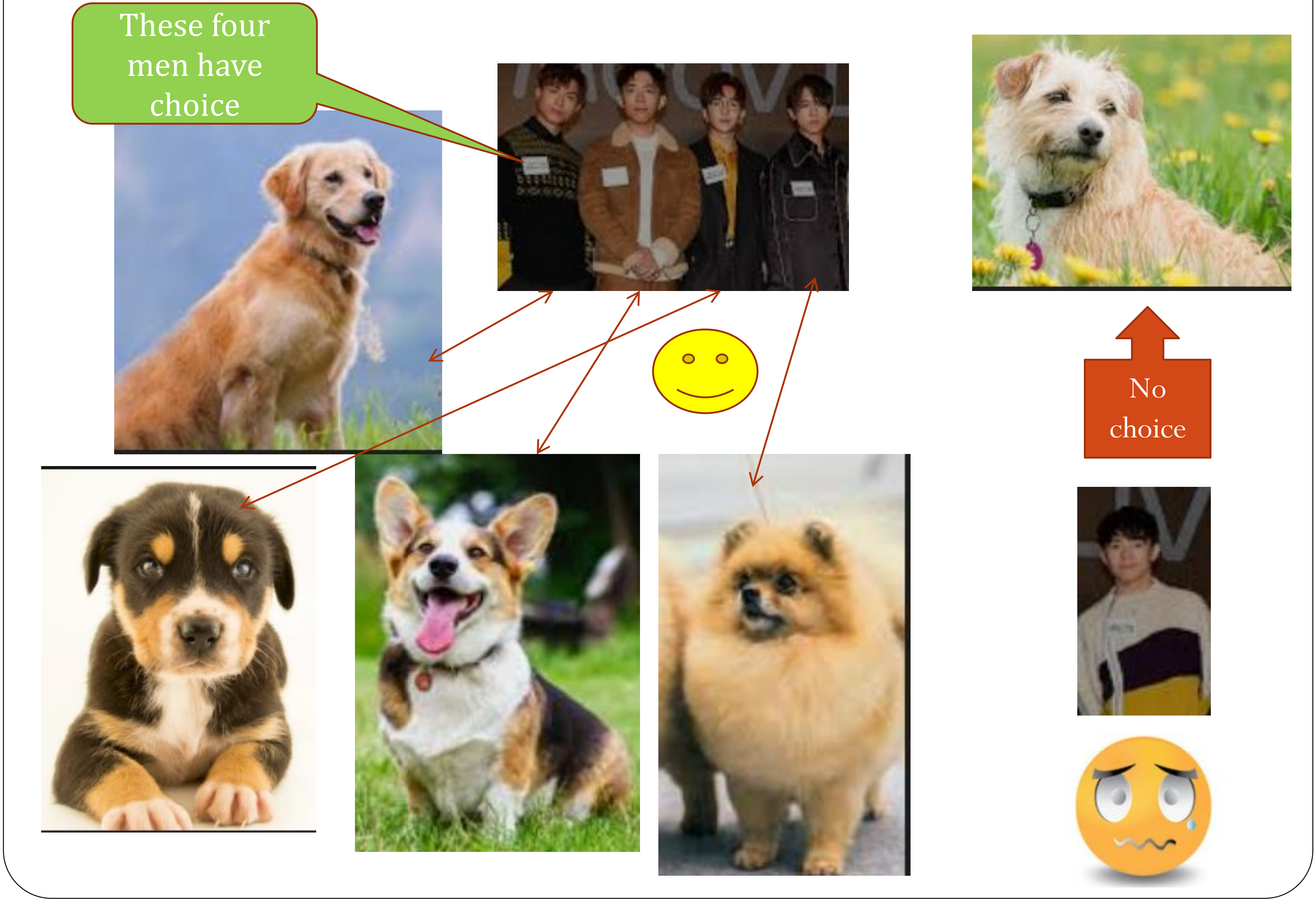
  通常 df=n-k。

其中n为样本数量，k为被限制的条件数或变量个数，或计算某一统计量时用到其它独立统计量的个数

OR

df = (number of rows-1)(number of column-1)

Degree of freedom自由度(df)



These four men have choice

No choice

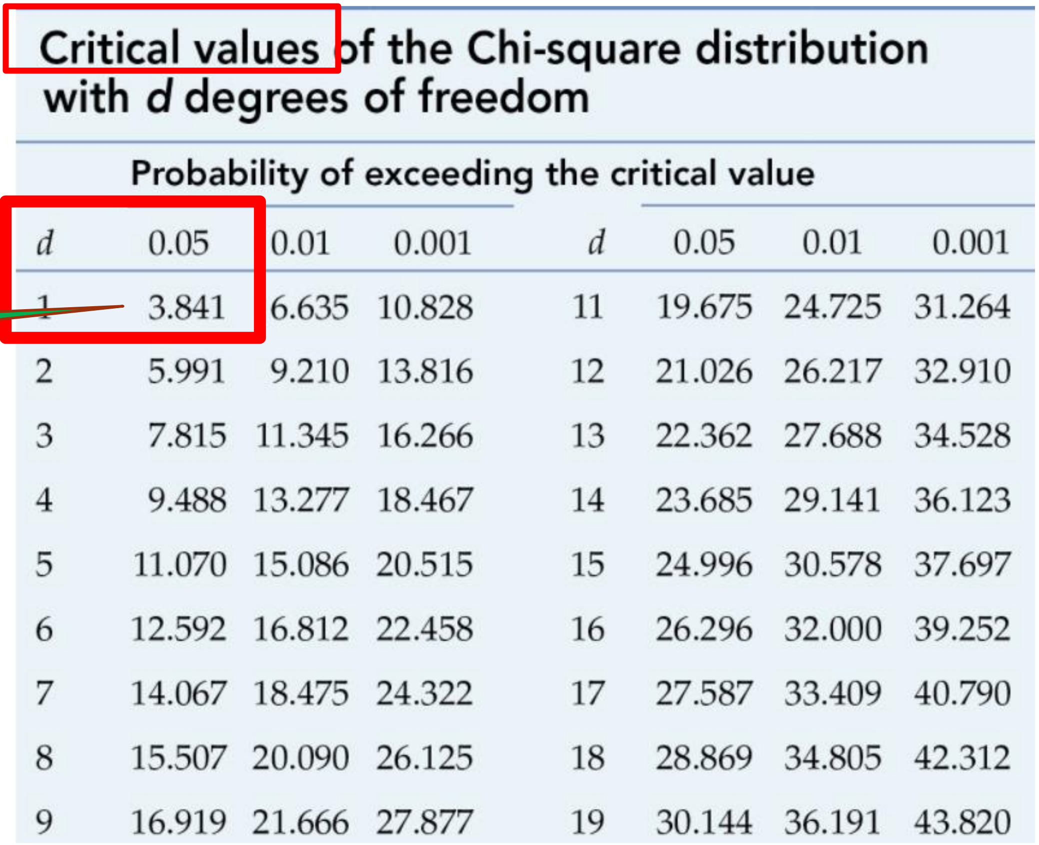
Calculate the degree of freedom 自由度

df=(number of rows -1)(number of columns-1)

df=(2-1)(2-1) = 1

	Dead	Alive
Drug A	a 30	b 40
Drug B	C 60	d 70

$X^2 = 0.198$



Critical values of the Chi-square distribution with *d* degrees of freedom

	Probability of exceeding the critical value						
d	0.05	0.01	0.001	*d*	0.05	0.01	0.001
1	3.841	6.635	10.828	11	19.675	24.725	31.264
2	5.991	9.210	13.816	12	21.026	26.217	32.910
3	7.815	11.345	16.266	13	22.362	27.688	34.528
4	9.488	13.277	18.467	14	23.685	29.141	36.123
5	11.070	15.086	20.515	15	24.996	30.578	37.697
6	12.592	16.812	22.458	16	26.296	32.000	39.252
7	14.067	18.475	24.322	17	27.587	33.409	40.790
8	15.507	20.090	26.125	18	28.869	34.805	42.312
9	16.919	21.666	27.877	19	30.144	36.191	43.820

X^2 < critical values

0.198 < 3.841

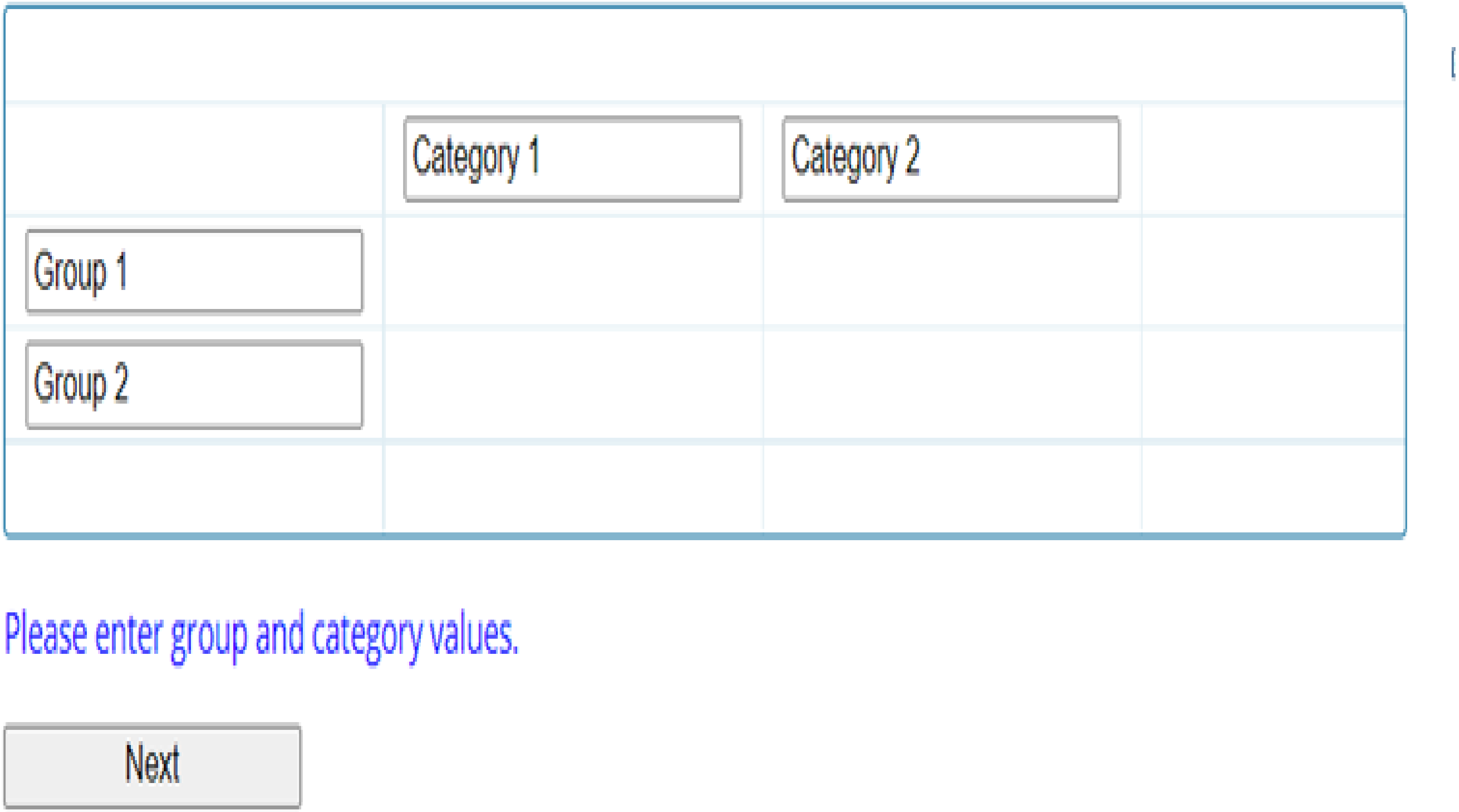
Do **NOT** reject

Null Hypothesis

不要拒绝零假设

https://www.socscistatistics.com/tests/chisquare/default2.aspx

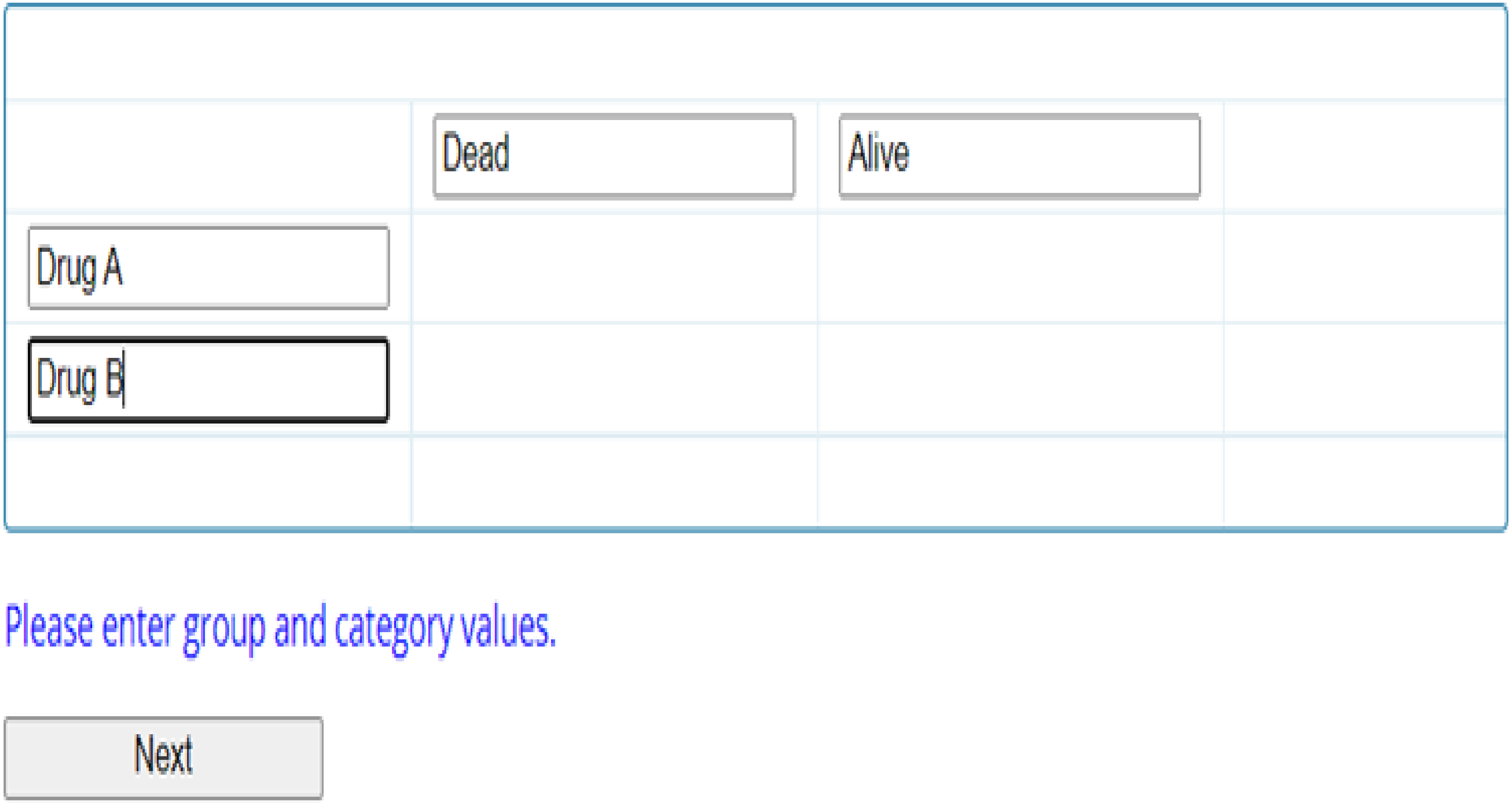
Use Chi Square Calculator



	Category 1	Category 2	
Group 1			
Group 2			

Please enter group and category values.

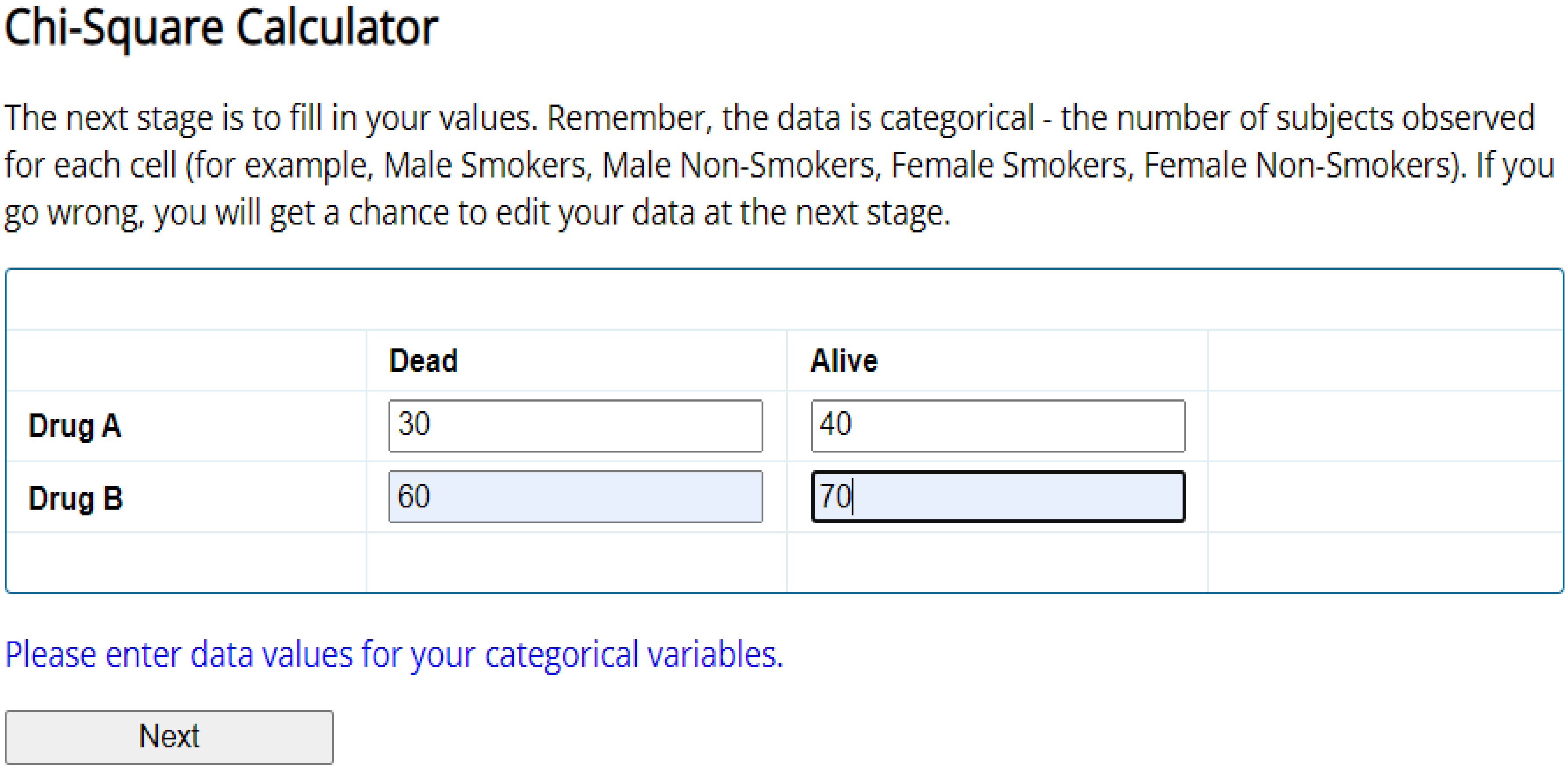
Step 1



	Dead	Alive	
Drug A			
Drug B			

Please enter group and category values.

Step 2



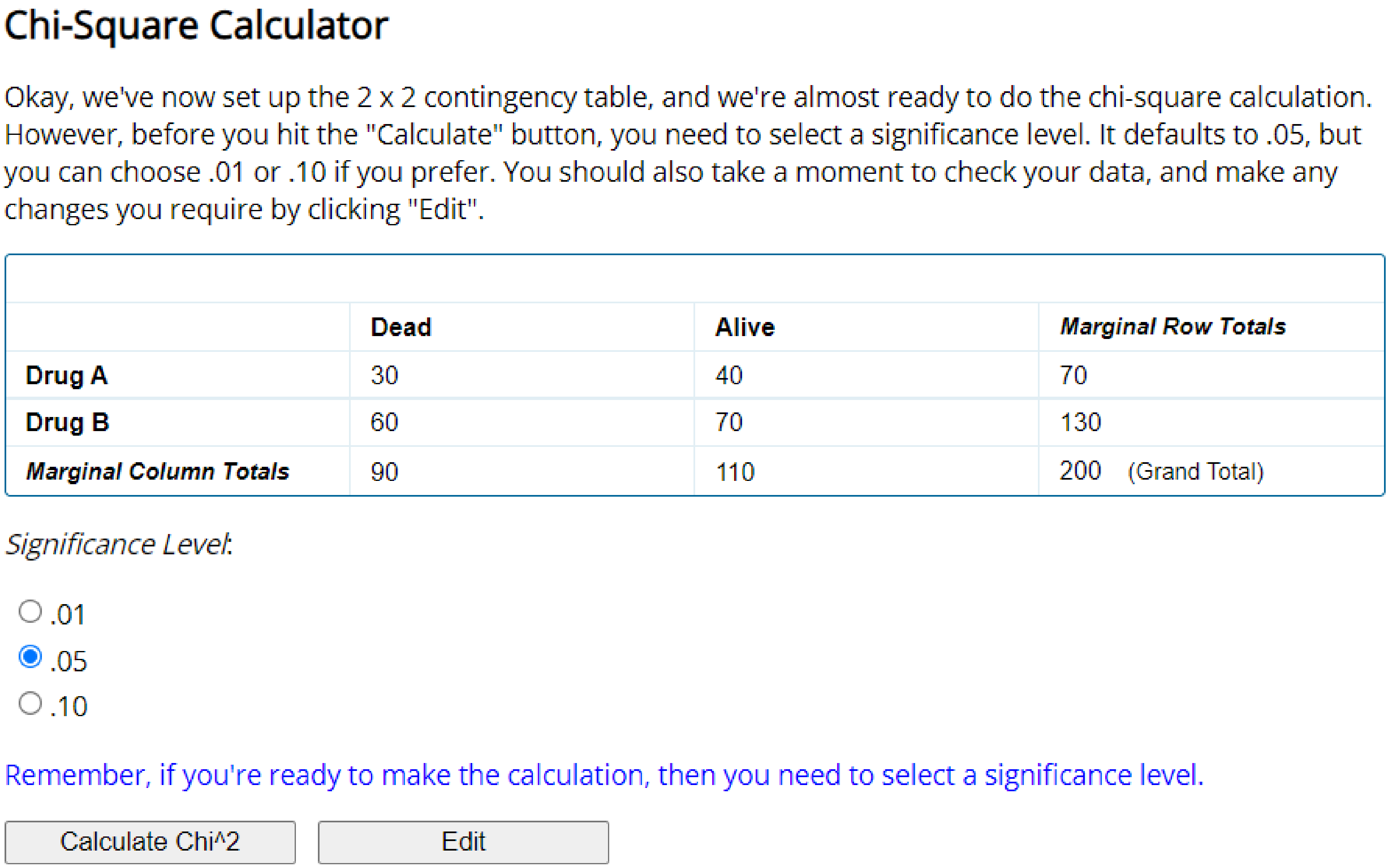
Chi-Square Calculator

The next stage is to fill in your values. Remember, the data is categorical - the number of subjects observed for each cell (for example, Male Smokers, Male Non-Smokers, Female Smokers, Female Non-Smokers). If you go wrong, you will get a chance to edit your data at the next stage.

	Dead	Alive	
Drug A	30	40	
Drug B	60	70	

Please enter data values for your categorical variables.

Step 3



Chi-Square Calculator

Okay, we've now set up the 2 x 2 contingency table, and we're almost ready to do the chi-square calculation. However, before you hit the "Calculate" button, you need to select a significance level. It defaults to .05, but you can choose .01 or .10 if you prefer. You should also take a moment to check your data, and make any changes you require by clicking "Edit".

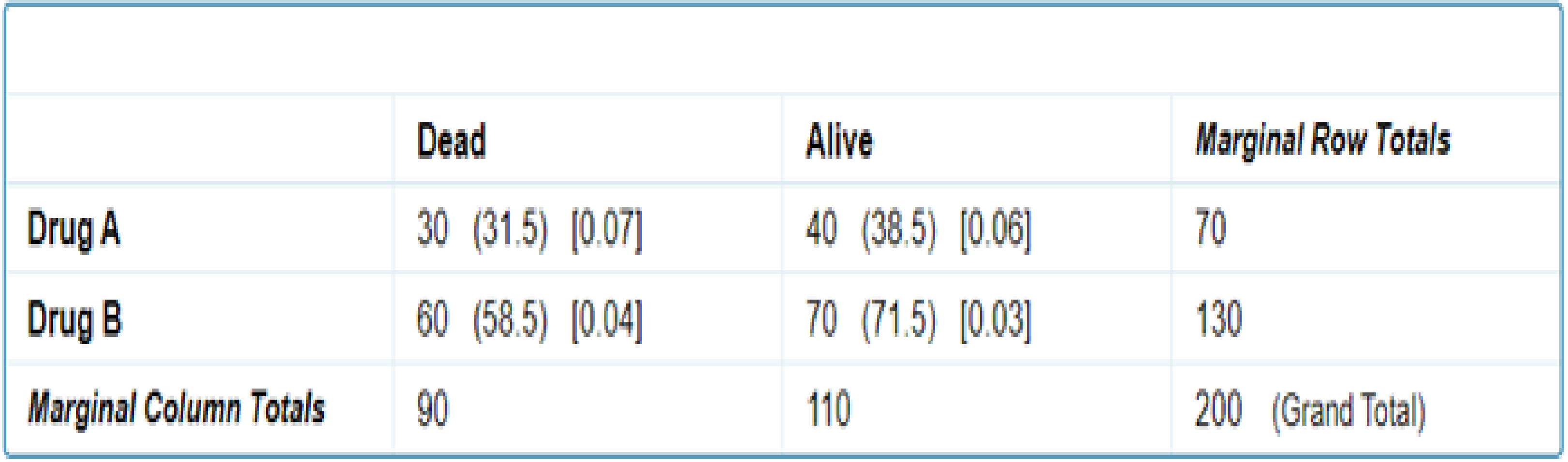
	Dead	**Alive**	***Marginal Row Totals***
Drug A	30	40	70
Drug B	60	70	130
Marginal Column Totals	90	110	200 (Grand Total)

Significance Level:

○ .01
● .05
○ .10

Remember, if you're ready to make the calculation, then you need to select a significance level.

Step 4



	Dead	Alive	*Marginal Row Totals*
Drug A	30 (31.5) [0.07]	40 (38.5) [0.06]	70
Drug B	60 (58.5) [0.04]	70 (71.5) [0.03]	130
Marginal Column Totals	90	110	200 (Grand Total)

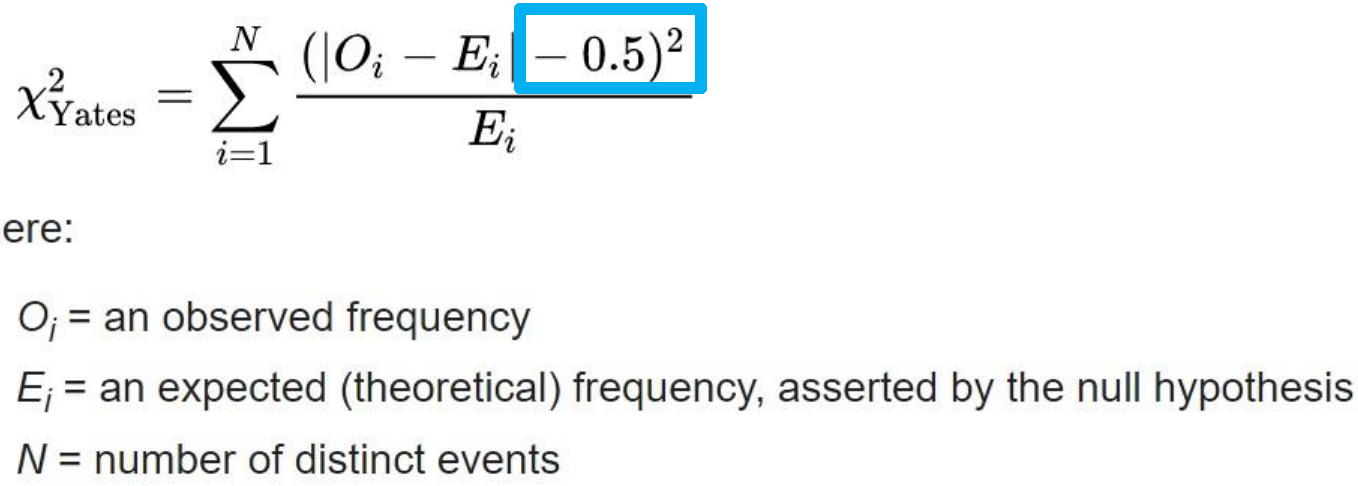
The chi-square statistic is 0.1998. The *p*-value is .654882. *Not* significant at $p < .05$.

The chi-square statistic with Yates correction is 0.0888. The *p*-value is .765708. *Not* significant at $p < .05$.

Yates correction 耶茨 修正

- Aims at correcting the error introduced by assuming that the discrete probabilities of observed binomial frequencies in the contingency table can be approximated by a continuous chi-squared distribution
- Also called the continuity correction for the chi-square test
- 卡方检验的连续性校正
- To adjust the observed frequency in each cell of a 2x2 table, Frank Yates suggested a correction by the following formula by subtracting 0.5 from the difference between each observed value and its expected value
- The correction is used only when there is one degree of freedom
- 修正仅在有一个自由度时使用

检定统计量：



$$\chi^2_{\text{Yates}} = \sum_{i=1}^{N} \frac{(|O_i - E_i| \boxed{- 0.5)^2}}{E_i}$$

where:

O_i = an observed frequency

E_i = an expected (theoretical) frequency, asserted by the null hypothesis

N = number of distinct events

X2 with Yates correction

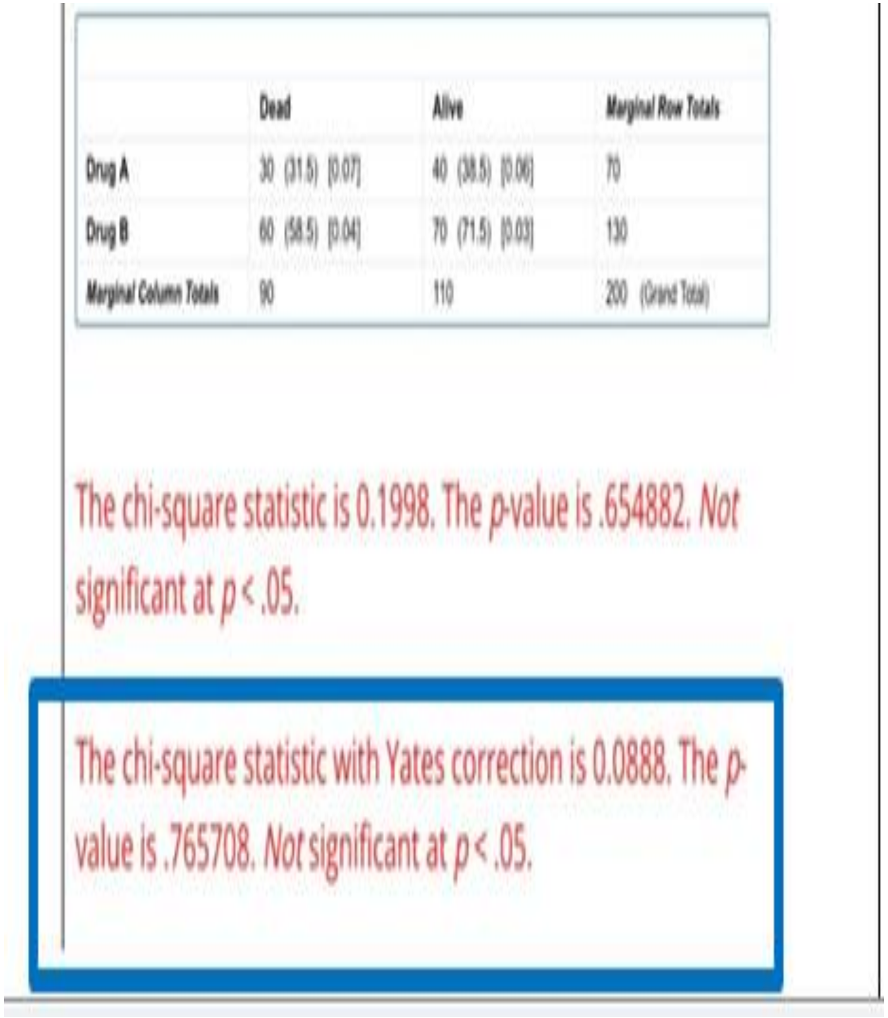
(use chi square calculator with Yates correction)

	Dead	Alive	
Drug A	a 30	b 40	70 (Row Total) a+b
Drug B	C 60	d 70	130 (Row Total) c+d
	90 (Column Total) a+c	110 (Column Total) b+d	200 Overall Total (a+b+c+d)

Use the same example

$X^2 = 0.198$

X^2 with Yates correction = 0.0888



	Dead	Alive	Marginal Row Totals
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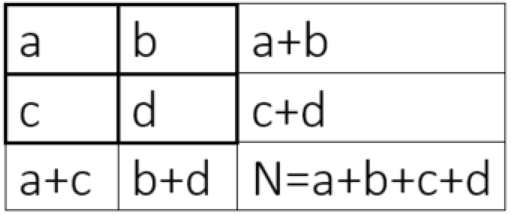
The chi-square statistic is 0.1998. The *p*-value is .654882. *Not* significant at *p* < .05.

The chi-square statistic with Yates correction is 0.0888. The *p*-value is .765708. *Not* significant at *p* < .05.

Fisher's Exact Test 确切概率法

- Another statistical significance test used in the analysis of contingency tables
- Significance of the deviation from a null hypothesis (e.g. *p*-value) can be calculated
- Employed when sample sizes/expected frequencies are small

对列联表进行关联性检定时，其方格内(如下表之 a、b、c、d)样本大小 n < 5，费氏精确检定法比较精准

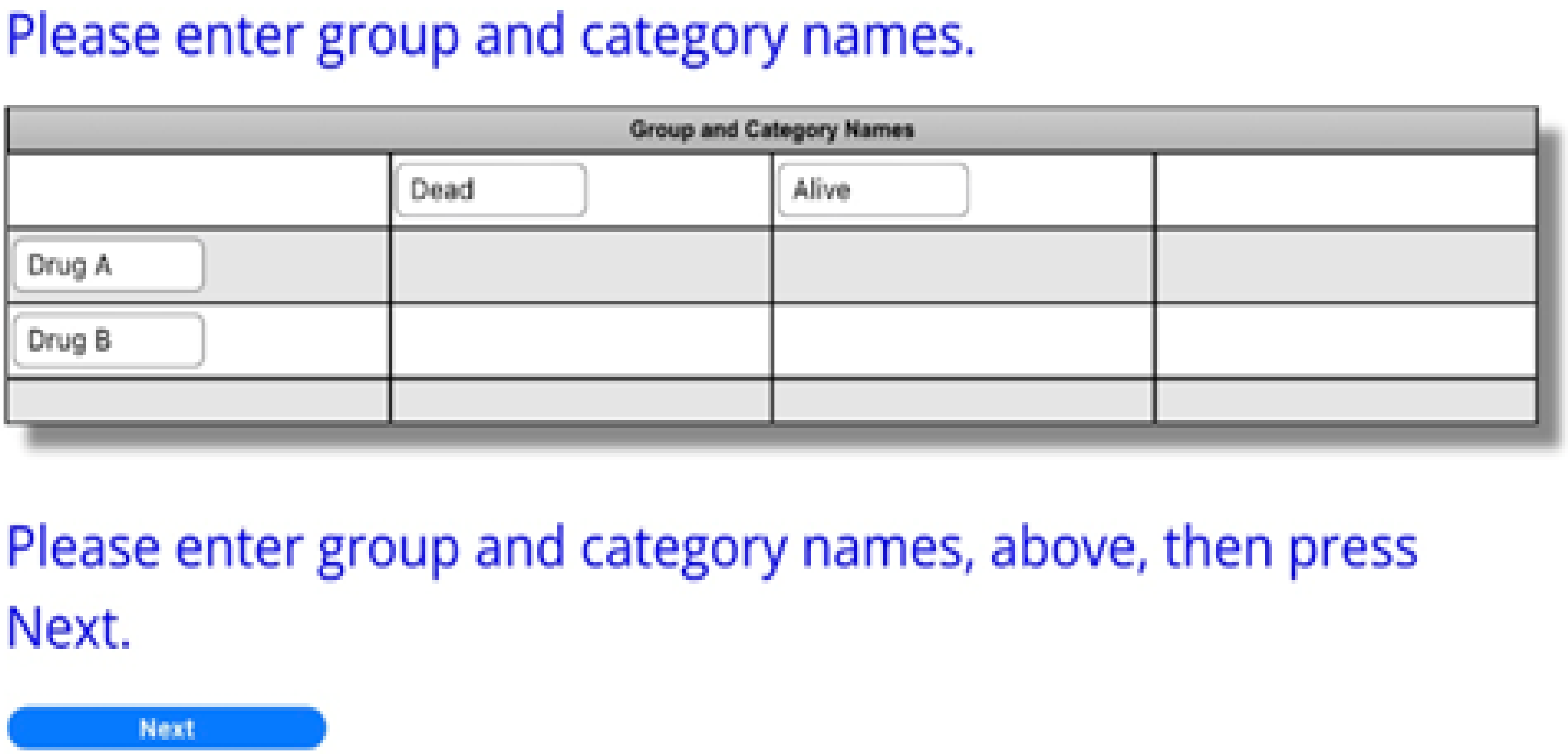


a	b	a+b
c	d	c+d
a+c	b+d	N=a+b+c+d

$$p = \frac{\binom{a+b}{a}\binom{c+d}{c}}{\binom{N}{a+c}}$$

Fisher’s Exact Test 确切概率法

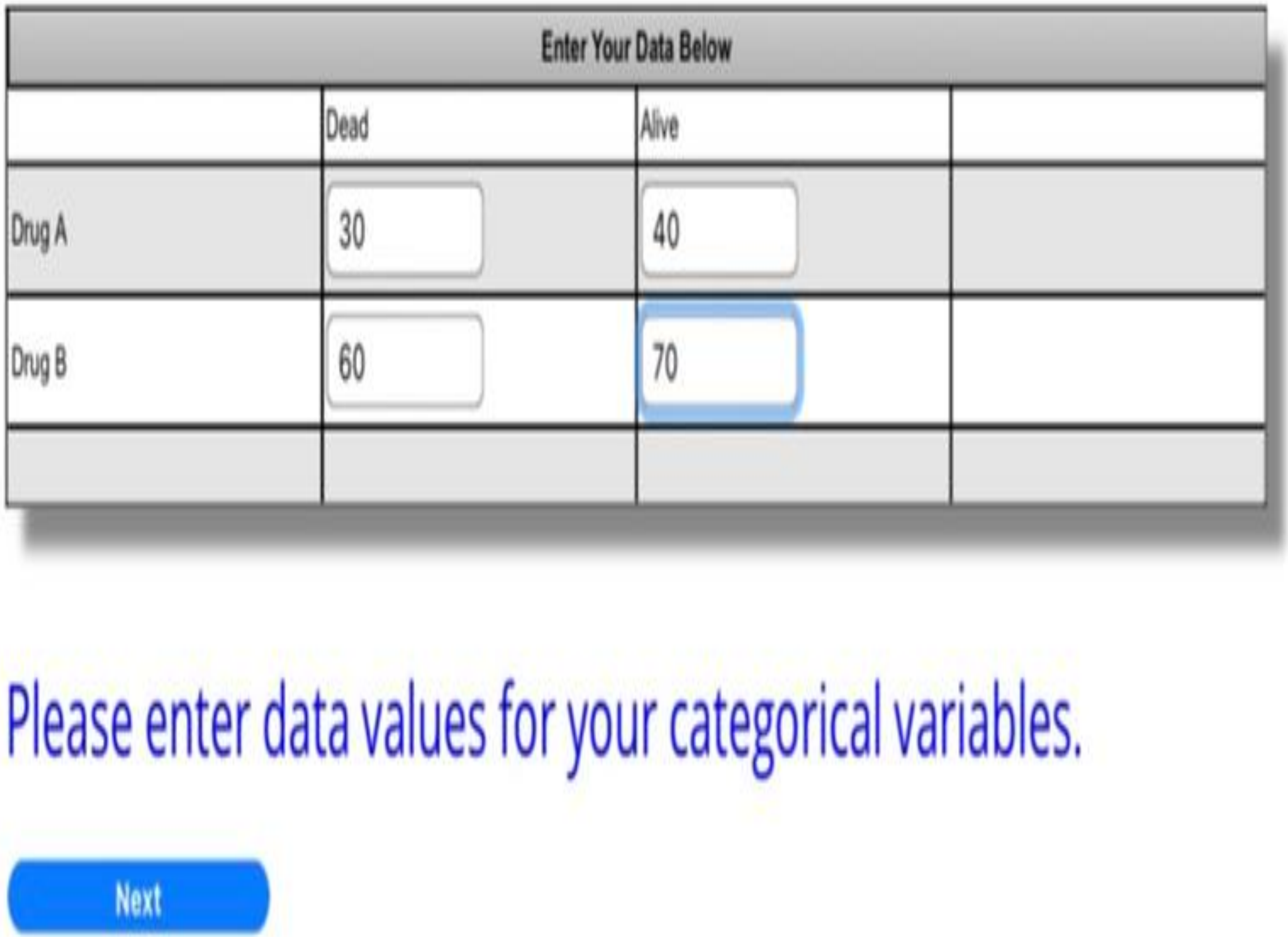
(use Fisher’s Exact Test calculator)



Please enter group and category names.

Group and Category Names			
	Dead	Alive	
Drug A			
Drug B			

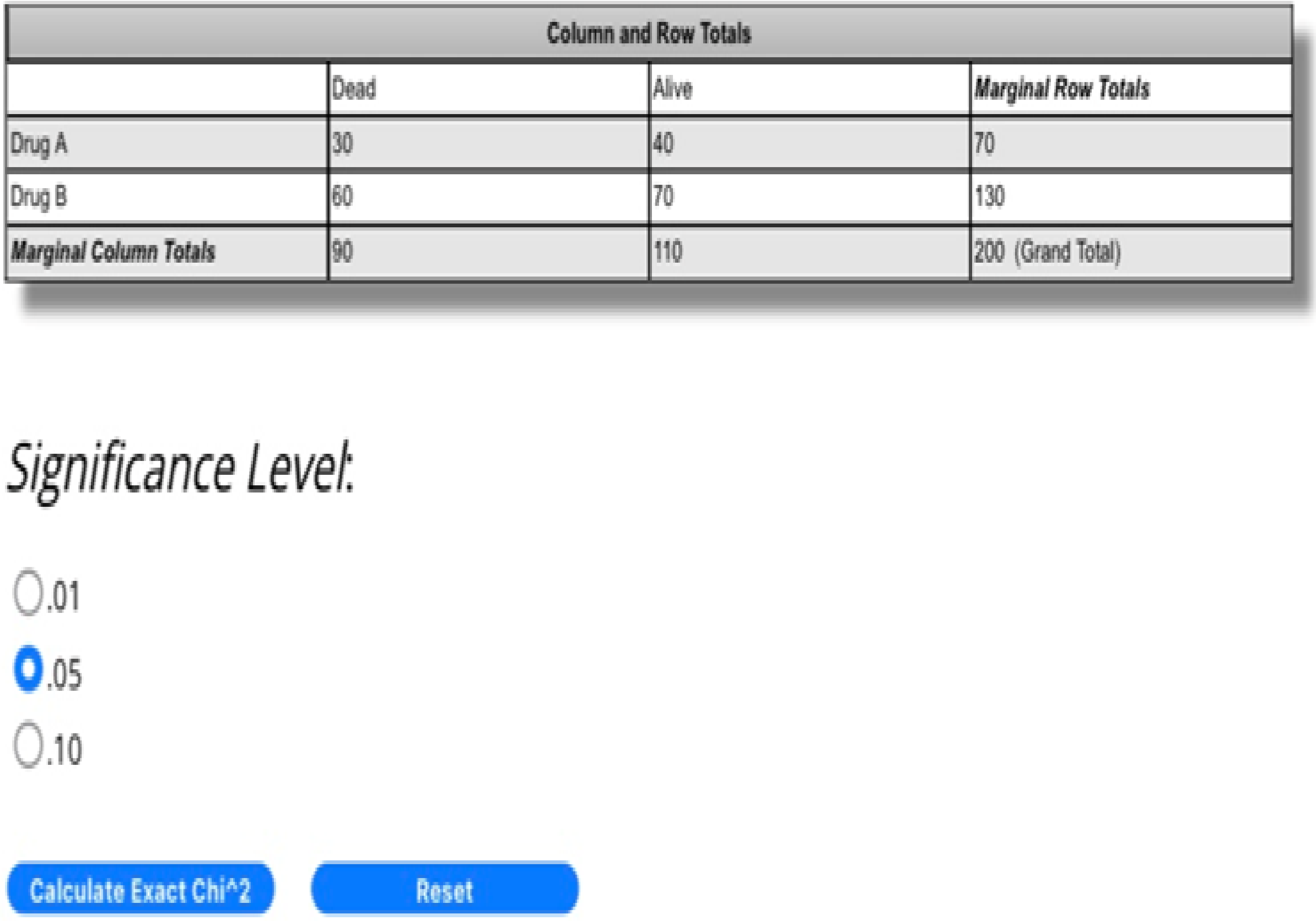
Please enter group and category names, above, then press Next.



Enter Your Data Below

	Dead	Alive	
Drug A	30	40	
Drug B	60	70	

Please enter data values for your categorical variables.



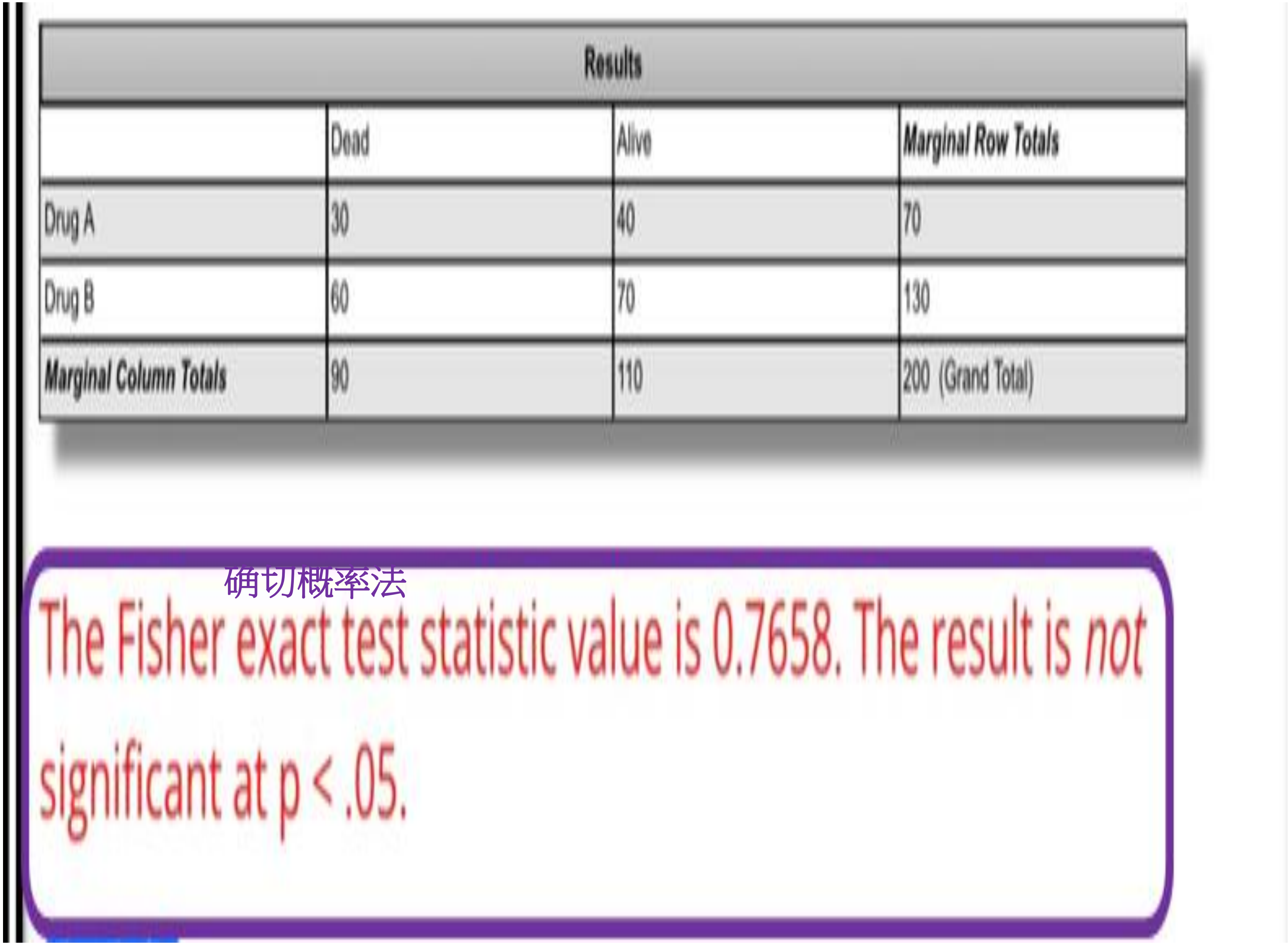
Column and Row Totals			
	Dead	Alive	***Marginal Row Totals***
Drug A	30	40	70
Drug B	60	70	130
Marginal Column Totals	90	110	200 (Grand Total)

Significance Level:

○ .01

● .05

○ .10

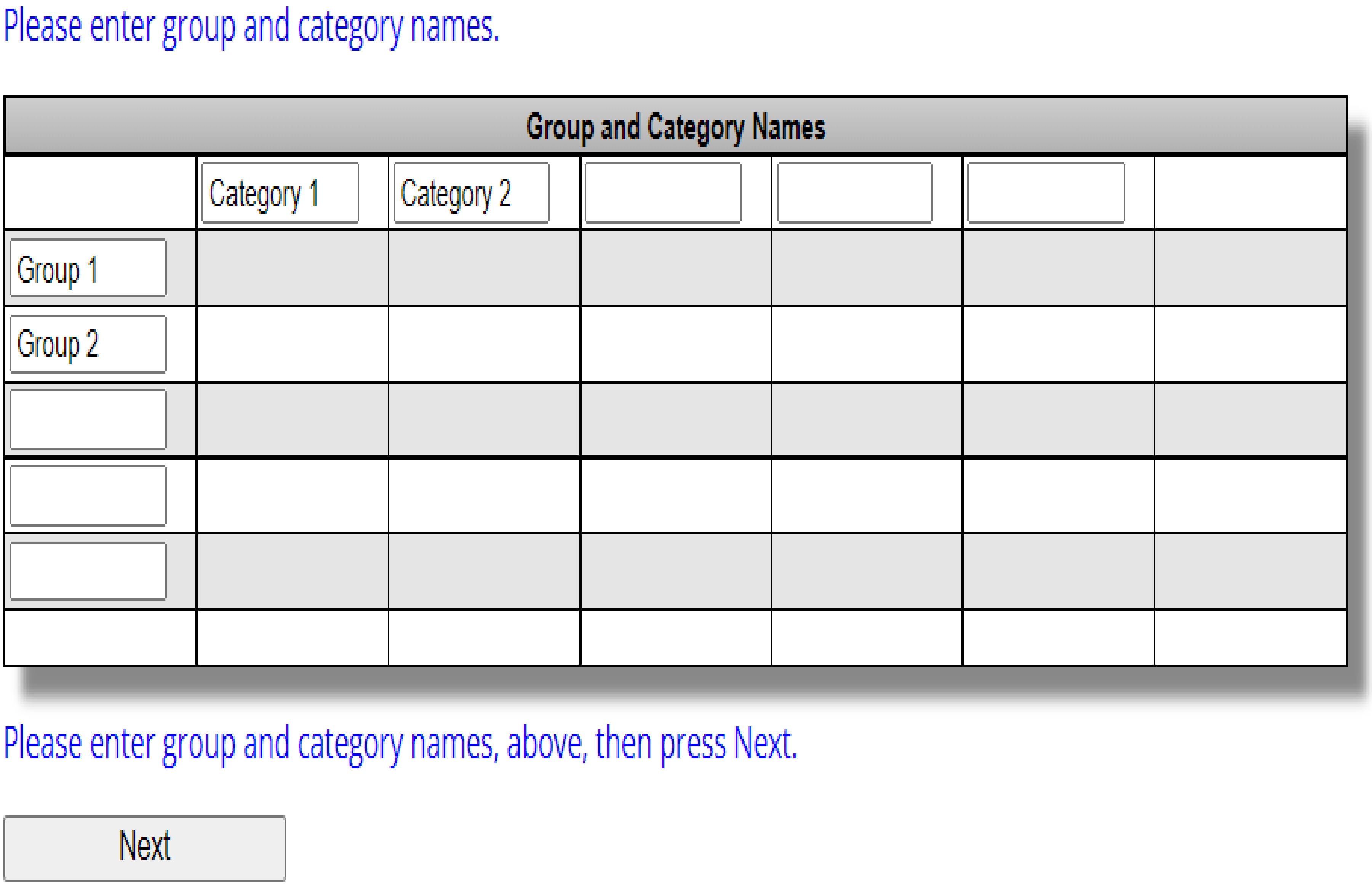


Results			
	Dead	Alive	*Marginal Row Totals*
Drug A	30	40	70
Drug B	60	70	130
Marginal Column Totals	90	110	200 (Grand Total)

确切概率法

The Fisher exact test statistic value is 0.7658. The result is *not* significant at $p < .05$.

r by c chi-square test 卡方分割（Calculator）



Please enter group and category names.

Group and Category Names						
	Category 1	Category 2				
Group 1						
Group 2						

Please enter group and category names, above, then press Next.

Thank You!!!